

EGM 3401

Theory Assignment #1

Spring 2015

Due Date: 17 February 2015 (In Class)

Question 1

Let \mathbf{u} be a vector with constant magnitude. Prove that

$${}^{\mathcal{A}} \frac{d\mathbf{u}}{dt} \cdot \mathbf{u} = 0$$

where \mathcal{A} is an arbitrary reference frame.

Question 2

Let ${}^{\mathcal{A}} d\mathbf{b}/dt$ be the rate of change of vector \mathbf{b} as viewed by an observer in reference frame \mathcal{A} . Explain why ${}^{\mathcal{A}} d\mathbf{b}/dt$ is observed the same in all reference frames.

Question 3

Let ${}^{\mathcal{A}} \boldsymbol{\omega}^{\mathcal{B}}$ be the angular velocity of reference frame \mathcal{B} as viewed by an observer in reference frame \mathcal{A} . Prove that ${}^{\mathcal{A}} \boldsymbol{\omega}^{\mathcal{B}} = -{}^{\mathcal{B}} \boldsymbol{\omega}^{\mathcal{A}}$.

Question 4

Let ${}^{\mathcal{A}} \boldsymbol{\omega}^{\mathcal{B}}$ be the angular velocity of reference frame \mathcal{B} as viewed by an observer in reference frame \mathcal{A} . Prove that

$${}^{\mathcal{A}} \frac{d}{dt} ({}^{\mathcal{A}} \boldsymbol{\omega}^{\mathcal{B}}) = {}^{\mathcal{B}} \frac{d}{dt} ({}^{\mathcal{A}} \boldsymbol{\omega}^{\mathcal{B}})$$

Question 5

Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a right-handed orthonormal basis fixed in reference frame \mathcal{B} . Prove that ${}^{\mathcal{A}} d\mathbf{e}_i/dt$, ($i = 1, 2, 3$) cannot have a component in the direction of \mathbf{e}_i , ($i = 1, 2, 3$) (that is, for any value of i , the rate of change of \mathbf{e}_i in reference frame \mathcal{A} cannot have a component in the direction of \mathbf{e}_i).

Question 6

Let \mathbf{u}_1 and \mathbf{u}_2 be orthogonal unit vectors. Prove that

$${}^{\mathcal{A}} \frac{d\mathbf{u}_1}{dt} \cdot \mathbf{u}_2 = -\mathbf{u}_1 \cdot {}^{\mathcal{A}} \frac{d\mathbf{u}_2}{dt}.$$

Question 7

Let \mathbf{a} and \mathbf{b} be vectors that lie in \mathbb{E}^3 . Furthermore, consider the scalar product $\mathbf{a} \cdot \mathbf{b}$ between the vectors \mathbf{a} and \mathbf{b} . Finally, let $\mathbf{E} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a basis for \mathbb{E}^3 . Show that the scalar product expressed in the basis \mathbf{E} can be written as

$$\{\mathbf{a} \cdot \mathbf{b}\}_{\mathbf{E}} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

where

$$\{\mathbf{a}\}_{\mathbf{E}} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \quad \text{and} \quad \{\mathbf{b}\}_{\mathbf{E}} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

are the matrix representations of the vectors \mathbf{a} and \mathbf{b} , respectively, in the basis \mathbf{E} .

Question 8

Let \mathbf{a} and \mathbf{b} be vectors that lie in \mathbb{E}^3 . Furthermore, consider the vector product between \mathbf{a} and \mathbf{b} . Finally, let $E = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a basis for \mathbb{E}^3 . Show that the vector product expressed in the basis E can be written in matrix-vector form as

$$\{\mathbf{a} \times \mathbf{b}\}_E = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$$

where

$$\{\mathbf{a}\}_E = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \text{and} \quad \{\mathbf{b}\}_E = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

are the matrix representations of the vectors \mathbf{a} and \mathbf{b} , respectively, in the basis E .

Question 9

Let \mathbf{a} and \mathbf{b} be vectors in \mathbb{E}^3 . Show that the triple vector product $\mathbf{a} \times (\mathbf{b} \times \mathbf{a})$ can be written as

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{a}) = (\mathbf{a} \cdot \mathbf{a})\mathbf{b} - (\mathbf{a}\mathbf{b})\mathbf{a}.$$